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Precision Broadband Wavemeter for Millimeter and Submillimeter Range

Y. A. Dryagin, V. V. Parshin, A. F. Krupnov, N. Gopalsami, and A. C. Raptis

***Abstract*— A precise, broadband, Fabry–Perot wavemeter has been designed and built to measure wavelengths in the millimeter and submillimeter range. The design of the wavemeter is novel in that it enhances the fundamental mode over a wide band and permits determination of the exact longitudinal index of the mode. With the use of an exact mode number in wavelength calculations, high measurement accuracies, to the extent permissible by the quality factor of the resonator, can be obtained. The wavemeter was tested by measuring well-known spectral lines of the OCS molecule in the frequency range of 72–607 GHz. Measurement of 24 OCS lines demonstrated an accuracy of better than 2×10^{-5} in relative units and 0.87×10^{-5} in rms units for frequency/wavelength. A discussion of further development and automation of the wavemeter is included.**

I. Introduction

In short-wave millimeter and submillimeter regions, open resonators of the Fabry–Perot type are analogs to closed cavities of the centimeter and millimeter wave regions [1]. They are based on concepts associated with optical frequencies and so are called quasi-optical Fabry–Perot resonators. The most common resonator employs a curved mirror at one end and a flat mirror at the other end. Stable Gaussian-beam resonances of the TEM_{mnq} type can be supported by these open resonators [2]. High quality factors on the order of 10^5 are routinely possible, which enable sharp resonances and high measurement accuracy of resonance locations.

Even so, the conventional method of measuring wavelength leads to diminished accuracy. It consists of tuning the high-Q quasi-optical Fabry–Perot resonator to two consecutive resonances (two consecutive longitudinal modes) and measuring the difference between the corresponding positions of a movable mirror. The difference is equal to half of the wavelength, with necessary diffraction corrections. The procedure is subject to two main sources of error.

1) The measured wavelength is the small difference between the two large distances (on the order of 100 mm) between the mirrors at the qth and $(q+1)$th longitudinal modes. The relative accuracy of measuring each resonance position is on the order of $1/Q = 10^{-5}$, but the relative accuracy of the difference in distance is on the order of $q/Q = 10^{-3}$.
2) If the oscillator whose radiation wavelength is to be measured drifts by 10^{-4} during the time the resonator is tuned from one mode to the another, the error in the wavelength measurement will be $q \times 10^{-4} = 10^{-2}$.

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Y. A. Dryagin, V. V. Parshin, and A. F. Krupnov are with the Institute of Applied Physics, Russian Academy of Sciences, 603600 Nizhnii Novgorod GSP-120, Russia.
N. Gopalsami and A. C. Raptis are with the Energy Technology Division, Argonne National Laboratory, Argonne, IL USA.

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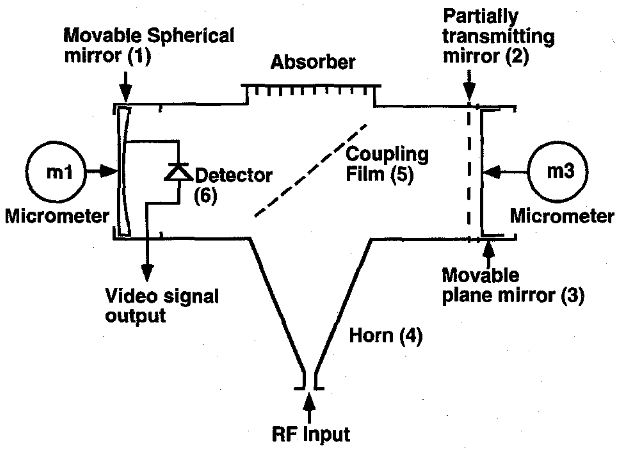

Fig. 1. Schematic diagram of a precision millimeter and submillimeter wavemeter.

Hence, it is desirable to tune the resonator only once; but, for this, it is necessary to know the exact index of the mode used. Also, it is necessary to separate the known (say, fundamental) mode from several higher order modes that can be present in a quasi-optical resonator. This paper describes a novel Fabry–Perot wavemeter design that eliminates the above sources of error and, at the same time, provides extremely broad-banded operation without the need for tuning and/or selection of the fundamental mode.

II. Design and Construction of a Novel Wavemeter

A schematic diagram of a new Fabry–Perot wavemeter is given in Fig. 1. The wavemeter contains two resonant cavities in tandem [3], [4]. A long cavity is formed between a movable spherical mirror (1 in the figure) with a radius of curvature R and a partially transmitting plane mirror (2) made of partially (1%) transparent metallized film. Behind the long cavity is a short cavity formed between the partially transmitting mirror (2) and a movable plane mirror (3). The wavemeter is fed by a horn (4) attached to the source, whose frequency/wavelength is to be measured. A beam splitter or coupling film (5) couples the radiation into the wavemeter cavities. The radiation transmitted directly through the beam splitter is either terminated into an absorber or used for a different application.

A built-in Shottky detector (6) is placed close to the center of Mirror 1. If the diode detector is placed near the center of Mirror 1, the response from the fundamental mode of the resonator is enhanced and responses from higher order modes are suppressed [5], [6]. Thus, one can distinguish between fundamental and higher order modes. Also, when the diode detector is near the center of Mirror 1, the working frequency range of the device is extremely broadened and such frequency-dependent details as detector housing and coupling elements (the coupling film chosen in the resonator, in principle, does not excite higher transversal modes) are avoided. For higher frequency stability, the body of the wavemeter is made of INVAR alloy, with coefficient of thermal expansion near zero at room temperature. A general view of the double-cavity wavemeter is shown in Fig. 2.

Key parameters that determine the cavity characteristics of a semispherical resonator are the Fresnel number N, stability factor g, quality factor Q, and the resonant wavelength λ, given by

$$N = \frac{a^2}{\lambda L} \tag{1}$$

$$0 \leq \left[g = \left\{1 - \frac{L}{R}\right\}\right] \leq 1 \tag{2}$$

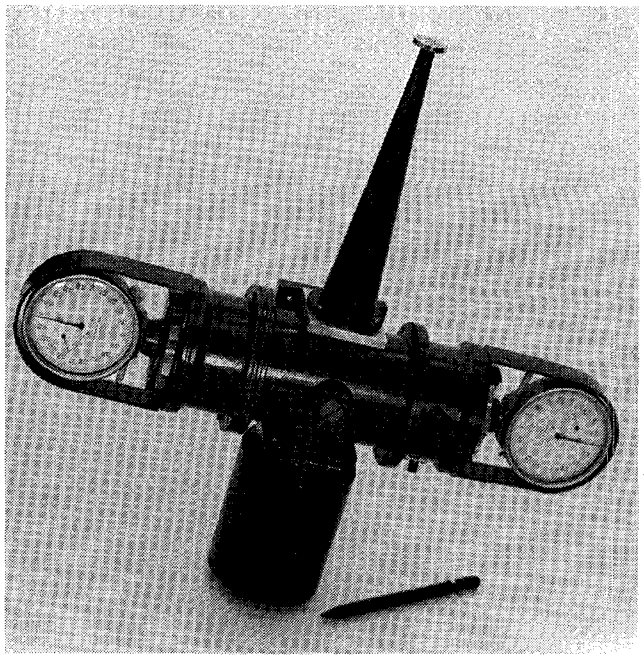

Fig. 2. Photograph of Fabry–Perot wavemeter.

$$Q = \frac{2\pi L}{\lambda\alpha} \tag{3}$$

and

$$\lambda = \frac{2L}{q + \pi^{-1} \arcsin\sqrt{\frac{L}{R}}} \tag{4}$$

where a is the radius of the cavity mirrors, λ is the wavelength of fundamental mode at resonance, L is the separation between the mirrors, R is the radius of curvature of the spherical mirror, and α is the resonator loss (fractional energy loss per transit), due to diffraction, output coupling, absorption, and scattering [2]. For the wavemeter that we built, $a = 6.3$ mm and $R = 300$ mm, and at a frequency of 150 GHz, $N = 5$, $g = 0.67$, $q = 100$, $L = 100$ mm, and Q is on the order of 10^5.

The procedure for measuring wavelength with the double-cavity wavemeter consists of first tuning the wavemeter to resonance between Mirrors 1 and 2 by moving spherical Mirror 1. Let $L = L_0 + \Delta L$, be the separation between Mirrors 1 and 2, in which L_0 is the initial path length between the mirrors when movable Mirror 1 touches the body of the wavemeter and ΔL is the displacement of Mirror 1 from its initial position for resonance on the fundamental mode. The position of resonance, ΔL is registered by a micrometer (m1 in Fig. 1). Then Mirror 3 is tuned to resonance between Mirrors 2 and 3, as seen by an increase of ≈20–50% in the resonance response. Let $l = l_0 + \partial l$, be the separation between Mirrors 2 and 3, in which l_0 is the initial length of the short resonator and ∂l is the displacement of Mirror 3 for resonance. The position of Mirror 3 is registered by a second micrometer m3.

From the short resonator formed by plane Mirrors 2 and 3, the resonant wavelength is approximately calculated from

$$l = \frac{\lambda}{2} q' \tag{5}$$

where q' is the longitudinal mode index of the short resonator. The value of q' is generally 1 or 2 for the first resonance when Mirror 3 is moved from its initial position (touching the body of the wavemeter). For longer-wave part of the working band ($\lambda \geq 1$ mm), $q' = 1$, and

TABLE I
MEASURED AND TABULATED OCS LINE FREQUENCIES

Tube Type	Tabulated line freq., MHz	ΔL, mm	q'	δl	Measured - Tabulated line freq. Δf, MHz	Relative Difference[a] x10^{-5}
OB-70	72 976.785	0.600	1	1.553	-0.7	-0.95
OB-71	85 139.109	2.011	1	1.260	0.2	0.23
	97 301.214	1.527	1	1.040	-0.3	-0.30
OB-86	109 463.069	1.150	1	0.870	1.2	1.10
	121 624.644	0.852	1	0.733	-1.5	-1.20
	133 785.905	0.604	1	0.621	1.6	1.20
	145 946.823	0.401	1	0.527	-0.2	-0.13
	158 107.366	0.228	1	0.447	-0.3	-0.18
	170 267.504	0.962	1	0.380	-0.8	-0.47
	182 427.203	0.775	1	0.321	0.0	0.00
	194 586.434	0.612	1	0.270	0.4	0.20
OB-30	279 685.307	0.411	1	0.035	-2.0	-0.70
	291 839.665	0.319	1	0.013	-2.5	-0.85
	303 993.273	0.234	2	0.485	1.2	0.39
	316 146.099	0.156	2	0.447	3.0	0.94
	328 298.114	0.086	2	0.413	-1.4	-0.42
	340 449.285	0.459	2	0.380	3.0	0.88
	352 599.581	0.384	2	0.351	-2.0	-0.56
	364 748.971	0.312	2	0.321	1.2	0.32
OB-80	510 459.600	0.325	2	0.087	7.7	1.50
	534 727.788	0.236	3	0.341	9.9	1.85
	583 247.415	0.086	4	0.528	-2.3	-0.39
	595 373.650	0.303	3	0.255	1.9	0.31
	607 498.353	0.266	5	0.734	10.9	1.79

[a]Relative difference between tabulated and measured frequencies

for shorter-wave part of working band, $q' = 2$. These cases are easily distinguishable from the wave band of the radiation source used (type of backward wave oscillator (BWO) in our case). Resonances with next higher values of q' also can be observed and distinguished. This tuning is much less critical (quality factor of the short resonator between Mirrors 2 and 3 is on the order of 1000) and the accuracy of this wavelength measurement is on the order of 10^{-3}. This accuracy is enough to distinguish among longitudinal modes and to remove ambiguity in the longitudinal mode number q of the long resonator between Mirrors 1 and 2. Using the value of λ calculated from (5) and knowing the values of L and R, one can calculate q from (4). The value of q obtained is rounded to the nearest integer and then substituted back in (4) to calculate λ. Thus, this procedure determines the exact longitudinal mode number, which in turn allows calculation of the wavelength at resonance accurately.

III. CALIBRATION OF WAVEMETER AND TEST RESULTS

The wavelength calculation in (4) does not take into account the refractive index of the medium (air) inside the wavemeter, the influence of thickness and refractive index of the coupling film, and imperfections in machining and mechanical tolerances of the wavemeter. Consequently, the wavemeter must be calibrated. The refractive index of air depends on humidity, temperature, and frequency band. Obviously, one can use a resonator under vacuum but doing so is much more complicated technically; a compromise would be to fill the cavity with dried air. The simplest way to remove these systematic errors is to calibrate the wavemeter by using well-known molecular absorption lines or a frequency synthesizer with adequate frequency resolution and stability [7].

A. Wavemeter Calibration

To calibrate the wavemeter in this work, we used a passive frequency standard in the form of a gas absorption cell. A 15-cm long sealed-off quartz cell [8], filled with OCS gas to a pressure of 5×10^{-2} Torr and fitted with Brewster angle windows, was placed between the horn and the wavemeter. A BWO connected to the horn supplied millimeter/submillimeter radiation. Using low-frequency modulation of the oscillator, both the wavemeter response and the gas absorption lines were observed simultaneously. If the wavemeter is tuned on the center of an OCS absorption line on an oscilloscope screen, the known line frequency can be used to calibrate the wavemeter.

If the wave velocity c is known, the quantity $\lambda = c/f$ can be calculated. For dry air, c = 299.712 km/s [9], [10]. If the air is at standard laboratory conditions (T = 20°C, P = 100,000 Pa, and

50% humidity), $c = 299{,}700$ km/s for frequencies <500 GHz. For frequencies in the 500–600 GHz range, the influence of water vapor lines is strong and if the air is not dried, measurements can either be inaccurate or cannot be repeated successfully.

The long and short resonators of the wavemeter are first tuned to a selected OCS line frequency f in the long-wave part of the working band ($\lambda \geq 1$ mm). Then, by moving the dial of Micrometer m3, its reading from (5) is set to $\partial l = \frac{c}{2f} - l_0$, where $l_0 = 0.5$ mm is chosen for convenience. The micrometer dial is fixed in this position for further measurements. Thus Mirror 3 is at a distance of $\approx\lambda/2$ from Mirror 2, and the reading of Micrometer m3 must be equal to $(\lambda/2) - 0.5$ mm. Then, substituting this value of λ and the mechanically measured or design values of L_0, ΔL, and R in (4), we can calculate the value of q and round it to the nearest integer. With the new value of q, (4) can be reused to find L_0 iteratively. By measuring several known frequencies, one can find the real optical (not geometrical) value of L_0. The tested wavemeter gave a value of 100.426 mm for L_0. This value, once calibrated, must hold good for further measurements.

B. Wavemeter Testing

Wavemeter tests were performed by measuring the positions of known spectral lines of the OCS molecule [11] in the range of 72–607 GHz. Backward wave oscillators OB-70 (52–70 GHz), OB-71 (78–119 GHz), OB-86 (118–178 GHz), OB-30 (258–375 GHz) and OB-80 (526–714 GHz) in free-running mode were used as the source of radiation. Using low-frequency modulation of the BWO, we tuned the wavemeter to resonance at selected OCS spectral lines. The wavelengths were measured and compared with the known frequencies of the OCS lines. For measured rotational lines of the OCS molecular spectrum (ground state, main isotope), experimental values of ΔL, q', ∂l, absolute difference Δf between measured and tabulated frequencies of lines, and relative difference in units of 10^{-5} are presented in Table I. In measurements obtained with oscillator OB-80, the resonator was hermetized and supplied with silica gel air dryer. As can be seen from Table I, the error in relative frequency/wavelength measurements over the whole band of 72–607 GHz did not exceed 2×10^{-5}. Rms relative error for 24 lines measured in the band was 0.87×10^{-5}.

IV. Conclusion and Further Development

A broadband Fabry–Perot wavemeter has been designed, built, and tested against known spectral lines of the OCS molecule. The wavemeter employs two cavities in tandem, which enables determination of the longitudinal mode index and calculation of the wavelength more accurately than possible with conventional methods. The wavemeter was tested without any changes for frequencies from 72 to 607 GHz. Highly accurate measurements were obtained in the whole band; the error of relative frequency/wavelength measurements was within 2×10^{-5}. The wavemeter is compact and rugged, and can be used for measurement of wavelengths in millimeter and submillimeter wave experiments. Applications of this device include calibration of millimeter and submillimeter BWO sources and frequency locking of BWO against the Fabry–Perot wavemeter response (or resonant line) by feedback control [12].

The wavemeter can be further improved by increasing the accuracy of mechanical movements and automation. Analog mechanical position measurement gauges (accuracy of 1μ, which for the length $L_0 = 100$ mm corresponds to a relative accuracy of 10^{-5}) can be replaced by more precise digital position measurement gauges. The calculations required to determine wavelength from the mechanical positions of the micrometers can be performed by a built-in microprocessor that processes the digital position data automatically and provides a digital display of the wavelength or frequency.

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